Time is a flat circle

An essay on eternal return, from Nietzsche's demon to a theorem in Lean

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Abstract. **[TODO: abstract.]** Four sentences when the essay is written. (1) The proposition: time is a flat circle, so every moment of a life returns, exactly and forever. (2) Two readings: an ethical test (Nietzsche) and a physical claim (Poincaré recurrence). (3) What a small Lean theorem shows about the physical reading, and what it cannot show. (4) The thesis the essay defends.

Keywords: eternal return; Nietzsche; Poincaré recurrence; Lean 4; philosophy of time.

1 Introduction

[SKELETON: this section states the proposition and the plan.]

The proposition: *time is a flat circle*. Everything that happens has happened before and will happen again, in the same order, without end.

The line is from the first season of *True Detective* (Noble 2014). The idea is older. Nietzsche used it as a test of a life (Nietzsche 1910).

To argue:

- The proposition has two readings. One is a claim about the world. The other is a question about how to live.
- The essay takes each reading in turn and asks what would make it true.
- Plan: Section 2 gives the ethical reading, Section 3 the physical one, Section 4 a formal statement, Section 5 the objections, and Section 6 the answer.

[TODO: an opening image or scene, 1 paragraph.]

2 The demon's question

[SKELETON: the ethical reading, from The Gay Science §341 and Zarathustra.]

To argue:

- In §341 a demon tells you that you will live this life again, many times, with nothing new. Nietzsche asks if you would curse him or bless him (Nietzsche 1910). **[TODO: quote §341 from the fetched text; the reference log marks it partial.]**
- The demon makes no claim about physics. The test works if the thought is only possible.
- In *Zarathustra* the thought first makes Zarathustra ill, and later he accepts it (Nietzsche 1999). **[TODO: cite the sections by name after reading them.]**
- Link to *amor fati*: to want nothing to be different.

[AUTHOR ANECDOTE: a moment you would choose to live again, or would not.]

3 The physicist's answer

[SKELETON: the physical reading. When does a system return to its start?]

To argue:

- Poincaré (1890) proved that a bounded system that keeps volume in its state space returns as close as you like to its start, and does so again and again (Poincaré 1890).
- This is a return *near* the start, not *to* it. With a finite number of states the return is exact (see Section 4).
- The return time for a real system is very long. **[TODO: give a number from a fetched source; do not estimate from memory.]**
- Nietzsche's notebooks try a similar argument from finite energy and infinite time (Nietzsche 1913). **[TODO: read §1066 and check what it says.]**
- **[TODO: Zermelo's recurrence objection to Boltzmann (1896). Fetch it before citing.]**

4 A formal circle

[SKELETON: the proposition as a theorem. The Lean file is `cycle/lean/Cycle/Clock.lean`.]

Model: a world has $n+1$ moments. One tick moves moment t to $(t+1)$ mod $(n+1)$. Lean proves four facts about this clock with no `sorry` and no Mathlib.

Table 1: The four theorems of `Cycle/Clock.lean`.

Theorem	Statement
`tick_iterate`	after k ticks, moment t becomes $(t+k)$ mod $(n+1)$
`eternal_return`	after $n+1$ ticks, every moment is itself again
`history_periodic`	the moment at time $k+n+1$ equals the moment at time k
`returns_forever`	every moment returns after $m(n+1)$ ticks, for every m

To argue:

- Table 1 makes the proposition exact, but the model assumes the circle. The proof shows that a circle returns. It does not show that time is a circle.
- The general result is stronger: any map on a finite set of states that is a bijection is periodic. **[TODO: prove it in Lean (pigeonhole on orbits), or say why it is left out.]**
- What a proof can carry into an essay, and what it cannot.

5 Objections

[SKELETON: the strongest cases against the proposition, one paragraph each.]

1. **Identity.** If no memory carries over, is the next "you" you? Ancient Stoics disagreed on this point. [**TODO: fetch a source on Stoic recurrence before citing.**]
2. **Entropy.** The second law gives time a direction. Recurrence and entropy growth seem to conflict. [**TODO: the Boltzmann–Zermelo answer, from a fetched source.**]
3. **Infinity.** A state space with infinitely many states need not return at all.
4. **Circle or spiral.** Perhaps events rhyme but do not repeat.
5. **Use.** If nothing can change, why choose? The answer from Section 2: the test asks you to choose as if it were true.

6 Conclusion

[**SKELETON: the answer to the proposition.**]

To argue:

- The physical claim is true only under strict conditions. Section 4 shows those conditions in their smallest form.
- The ethical claim needs none of them. It asks for a life you would accept on repeat.
- [**TODO: thesis, one sentence. Draft: "Time is probably not a flat circle, but you should live as if it were."**]

Statements

AI use. An AI assistant (Claude) wrote this skeleton: the section plan, the notes and the Lean file. The author writes the prose.

References

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